

# **3D Time Dependent Stokes Vector Radiative Transfer in an Atmosphere-Ocean System Including a Stochastic Interface**

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## **LONG-TERM GOALS**

The major objective of this proposal is to calculate the 3-D, time dependent radiation field both within the ocean and in the atmosphere in the presence of a stochastically varying interface which may also be perturbed by sea foam, air bubbles, surfactants, rain, etc. This study will serve as the genesis to the future evolution of an inversion algorithm whereby one could reconstruct images that have been distorted by the interface between the atmosphere and the ocean or the ocean itself. This study will rely heavily on both the spectral and polarimetric properties of the radiation field to deduce both the sea state and the perturbations produced on it. A second phase of this study will be to explore the asymptotic polarized light field and to determine how much information can be obtained about the IOP's of the medium by measuring it. The third phase of this proposal will deal with the problem of improving image resolution in the ocean using some novel polarimetric techniques that we are just beginning to explore. Once these studies have been completed using a passive source, it will be rather straightforward to extend them to active sources where we can explore the use of both photo-acoustic and ultrasound-modulated optical tomography to improve image resolution.

## **OBJECTIVES**

The new Navy initiative is focusing on one of the most formidable problems in radiative transfer theory; namely, calculating the full 3D time dependent radiation field (with full Mueller matrix treatment) in a coupled atmosphere-ocean system where the boundary separating the two has both spatial and temporal dependence. Although a great deal of work has been done on obtaining power spectra for ocean waves, I know of no work that has yielded similar results for the radiation field within the ocean. It is clear that as long as the surface has a significant effect on the internal light field, it will leave its signature on the radiation field within the ocean and the relative strength of this field compared to the ambient field will determine the success or failure of inversion algorithms. However, as we go deeper within the ocean we start to enter a region called the asymptotic region where all photons lose memory of their origin and the light field then remains stationary and becomes independent of the azimuthal angle. The depth dependence becomes simply exponential, i.e.  $\mathbf{L}(z+h, \theta) = \mathbf{L}(z, \theta) \exp(-kh)$  where  $k$  is called the diffusion exponent. It should be noted at this juncture that this asymptotic light field is still polarized which is why we used the bold-faced vector notation. We were the first to compute the degree of polarization for this asymptotic light field for Rayleigh scattering and were able to obtain an analytic expression for both the polarized radiation field and the diffusion exponent (see ref. 1). In addition, we were also able to set up a numerical scheme to compute the polarized radiation field as well as the diffusion exponent for any single scattering Mueller matrix.

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The interesting feature about the asymptotic light field is that it depends profoundly on both the single scattering albedo as well as the phase function of the medium. We also found that substantial errors will occur in both the ordinary radiance and the diffusion exponent if they are calculated from scalar rather than vector theory

## APPROACH

There are several stages to our approach that we will enumerate. The sine qua non for this entire project will be the development of a fully time dependent 3-D code capable of calculating the complete radiation field, i.e. the complete Mueller or Green matrix at any point within the atmosphere-ocean system. This of course implies that both horizontal as well as vertical IOP's must be accounted for. It should also be noted that the code must be capable of handling internal sources as well in order to explore both fluorescence and bioluminescence. At present there are several 3D codes that are able to compute various radiometric quantities in inhomogeneous media; however, as far as we know, none exists which will couple both atmosphere and ocean with a time dependent stochastic interface. One of the earliest 3D radiative transfer (RT) codes was developed by Stenholm, et. al<sup>2</sup> to model thermal emission from spherical and non-spherical dust clouds. It was based on an implicit discretization of the transfer equation in Cartesian frames. To our knowledge, the first 3D-scalar RT code using discrete ordinates was written by Sánchez<sup>3</sup> et al.; however, it did not make use of spherical harmonics and lacked efficiency and accuracy particularly for small viewing angles. The addition of polarization to the 3D discrete ordinates method was done by Haferman<sup>4</sup> et al. Almost concurrently, a 3D-scalar RT code was written by K. F. Evans<sup>5</sup> which used both spherical harmonics and discrete ordinates. This method uses a spherical harmonic angular representation to reduce memory and CPU time in computing the source function and then the RT equation is integrated along discrete ordinates through a spatial grid to model the radiation streams. We have already obtained this code and will use it for validation of our 3D scalar Monte Carlo code for both the atmosphere and ocean components. Several Monte Carlo codes both scalar and vector have been published for solving specialized problems in atmospheric optics usually dealing with finite clouds<sup>6,7,8,9</sup>. Without exception, these codes are using quite primitive, also called "brute force", methods. None of them will do what we are proposing in our approach to the fully time dependent 3D solution applicable to both atmosphere and ocean. It should be mentioned that we have already successfully added to our Green matrix Monte Carlo code the capability to handle internal sources such as fluorescence, bioluminescence and even thermal emission.

Once we have developed our 3D code to handle both the ocean and atmosphere without the interface included, we will then develop a 3D form of 1D matrix operator theory that we worked out in two seminal papers published in Applied Optics<sup>10,11</sup>. The basic idea of the method is that if one knows the reflection and transmission operators of say two layers, then it is rather straightforward to get the reflection and transmission operators for the combined layer. With this method we can start from an infinitesimal layer and build large and even semi-infinite layers in a rapid way, i.e. if we start with a layer of thickness  $\Delta$  then in  $N$  steps we can reach a thickness of  $2^N \Delta$ . Another very relevant feature of this method is that it will allow us to add the interface to the "bare" ocean, i.e., one without an interface, to get the combined ocean-interface operator and then add this layer to the atmosphere for the final reflection and transmission operators for the combined system. The question immediately arises is why not solve the entire system at the same time? The answer is that by doing it this way we only have to use the adding feature to combine the time dependent interface thus avoiding performing the entire calculation at each instant in time. This method has also become known as the adding-doubling method. These operators are effectively the impulse response or Green matrix for the upper

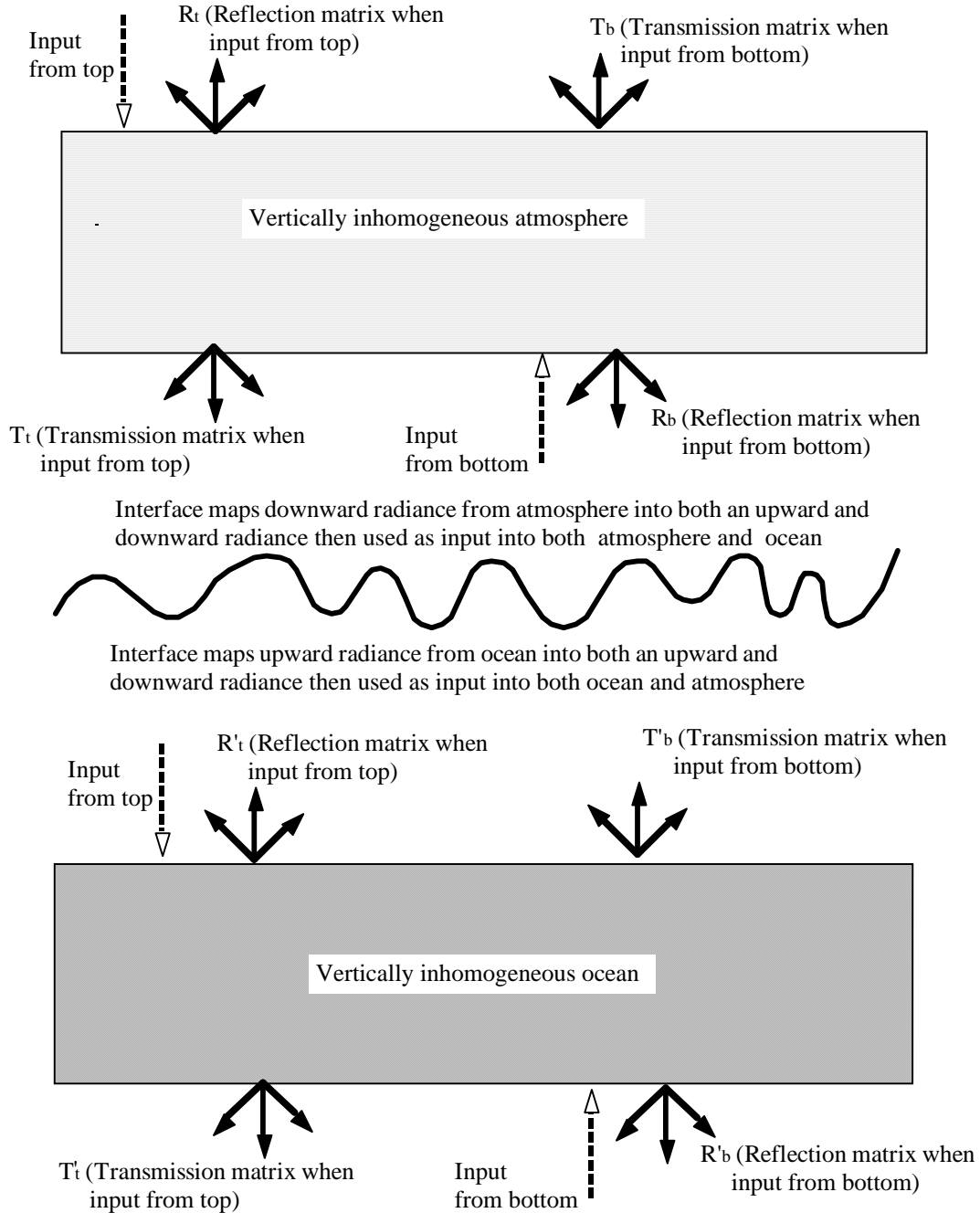
and lower boundaries of the medium. Therefore, if we know the external radiance input into both upper and lower layers, we can then obtain the output at both the upper and lower boundaries of the combined system. A pictorial description of the method is shown in Fig. 1. It should be emphasized that this method will also handle internal sources as well such as bioluminescence, fluorescence, and even thermal emission. This method can also handle detectors at any interior point in the medium. Another bonus of matrix operator theory is that one can easily obtain the path radiance between source and detector which is a sine qua non for image analysis.

In order to add a interface which is spatially inhomogeneous in the  $y$  direction but homogeneous in the  $x$  direction, we will need the reflection (R) and transmission (T) operators for both the atmosphere and ocean now as a function of time  $t$  and both  $z$  and  $y$ ; namely  $R(y-y_0, z, t, \theta, \phi)$  and similarly for  $T$ . It is important to note that we only have to obtain the response of the atmosphere or ocean to a single line source at the point  $y_0$  and then using the translational invariance of the medium in the  $y$ -direction will have the reflection and transmission operators at every point in the  $y$ -direction. The only method we know to create these 2-D operators is Monte Carlo. Once these are obtained, we can use the output from each layer as the input to the surface boundary whose reflection and transmission properties are either known or calculable. For instance if the surface consists of just capillary or gravity waves, then we just need the Fresnel coefficients to give us the requisite reflection and transmission operators for the interface. Now once these operators are obtained then we can use extended matrix operator theory to get the final time dependent radiation field that a detector will see. Let us consider the simplest case where the surface is 1-D and we know its power spectrum. It should be emphasized at this point that it is not sufficient to have just a wave-slope distribution since it will only give us statistically averaged results for the radiance field. The introduction of the spatial and temporal dependent interface destroys the symmetry and makes all 1-D codes essentially useless in this domain. At each instant in time, the surface will have a distinct shape that will evolve in time. We have developed a method using linear filter theory whereby we can take an ocean power spectrum and using a random number generator create a realistic surface that will match the original power spectrum and will still exhibit both stationarity and ergodicity. Now the nice feature about what we are proposing is that we can now concentrate on just the effects of the surface on the detectors since as the surface evolves in time so too will the radiance field as recorded by the detectors. Now both the spatial and temporal profiles will be constantly changing; however, we will have created them from a medium which has been assumed stationary and only the interface produces the time dependence and the horizontal spatial inhomogeneities, i.e. the R and T operators for both the atmosphere and ocean need only to be computed **once**. This is clearly a first-order solution to the more complex problem; however, it should tell us a great deal about future complexities of inversion and also the efficacy of pursuing the next level of difficulty. If the surface is perturbed by foam, bubbles, etc. then these can be added and the matrix operator theory will be used to calculate the effective reflection and transmission operators of the perturbed surface. It should also be stated that this project is enormously computationally intensive; however, the type of codes we will produce are ideally suited for large-scale parallel processors, which we do have access to.

The next level of difficulty is where we will use Monte Carlo methods to compute a full 3-D distribution of the time dependent radiation field, which now may include 3-D inhomogeneities in both the ocean and atmosphere. This will be a computational tour de force requiring a major new computer program that must be capable of placing IOP's of both the atmosphere and ocean at each point in a large 3-D grid. Matrix operator theory will again be used but it will now be much more complicated since our reflection and transmission operators now become functions of three spatial variables. In fact, the complete solution to this problem could approach the complexity of the general circulation

models used in weather forecasting. Due to the large volume of data that will be generated, we will clearly have to develop methods to easily display animated sequences of this time dependence.

These projects were worked on by Dr. Pengwang Zhai, who left for NASA Langley Research Center in August of 2008, and are now being worked on by Dr. Yu You, a postdoctoral research associate.



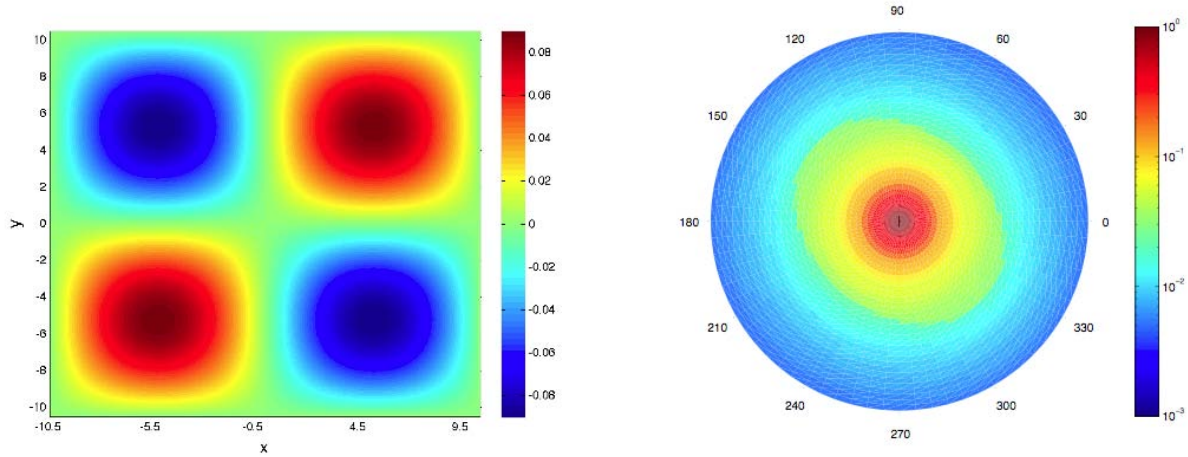
**Fig. 1. Schematic representation on the use of matrix operator theory to calculate the time dependent radiation field within the ocean**

## WORK COMPLETED

- a) We have completed a study of single scattering properties of tri-axial ellispoidal particles for a size parameter range from the Rayleigh to geometric-optics regimes, using both the discrete-dipole approximation (DDA) method and the geometric optics method (GOM). A paper on this work has been submitted to Applied Optics.
- b) We have applied the Monte Carlo code to studies of polarization effects on upwelling marine atmospheric radiances, which play important roles in the remote sensing of ocean color using the future National Polar-orbiting Operational Environmental Satellite System (NPOESS) satellite, especially in the validation of the associated ocean color retrieval algorithms. A paper on this work has been submitted to Applied Optics.
- c) We have completed a study of using the Monte Carlo code to investigate the Mueller matrix imaging of targets under an air-sea interface. A paper on this work has been submitted to Applied Optics.
- d) We have developed a hybrid matrix operator-Monte Carlo (HMOMC) method to solve the vector radiative transfer equation (VRTE) in a 3D atmosphere-ocean system (AOS). The primary advantage of this hybrid method is that it solves the VRTE efficiently in an AOS with different dielectric interfaces but with the same atmospheric and oceanic conditions. Two papers on this work have appeared in Applied Optics, 47, pp. 1037-1047 and pp. 1063-1071 (2008).
- e) We have applied the pseudo-spectral time domain (PSTD) method to calculations of single scattering properties of nonspherical particles. In this study, we show that the scattering phase functions computed from PSTD method agree with the counterparts computed from the finite-difference time-domain (FDTD) method for nonspherical particles with size parameter of up to 20. A paper on this work has appeared in JOSA A, 25, 785-790 (2008).
- f) We have completed a study of the relationship between lidar backscatter and the corresponding depolarization ratio for various cloud systems using satellite data collected by the Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observations (CALIPSO). A paper on this work has appeared in Optics Express, 16, 3931-3948, (2008).
- g) We have generalized the DDA method and applied it to the confirmation of the efficacy of invisibility cloaks made from metamaterials. In this study, we generalized the current DDA formalism to situations where the scattering object under investigation has not only electric response, but also magnetic response, to an incident electromagnetic wave. We then derived the material properties for invisibility cloaks with irregular shapes and confirmed that the designated properties lead to cloaking effects. Two papers on this work have appeared in Optics Express, 16, 2068-2079 and 6134-6145, (2008).
- h) We have completed a study of the effects of surface roughness of ice particles on the retrieval of the optical and microphysical properties of ice clouds from satellite observations. In this study, we show that a simplified ray-tracing computational scheme can account for most of the effects of surface roughness on particle single-scattering properties. A paper on this work has appeared in IEEE Transactions on Geoscience and Remote Sensing, 46, 1940-1947, (2008).

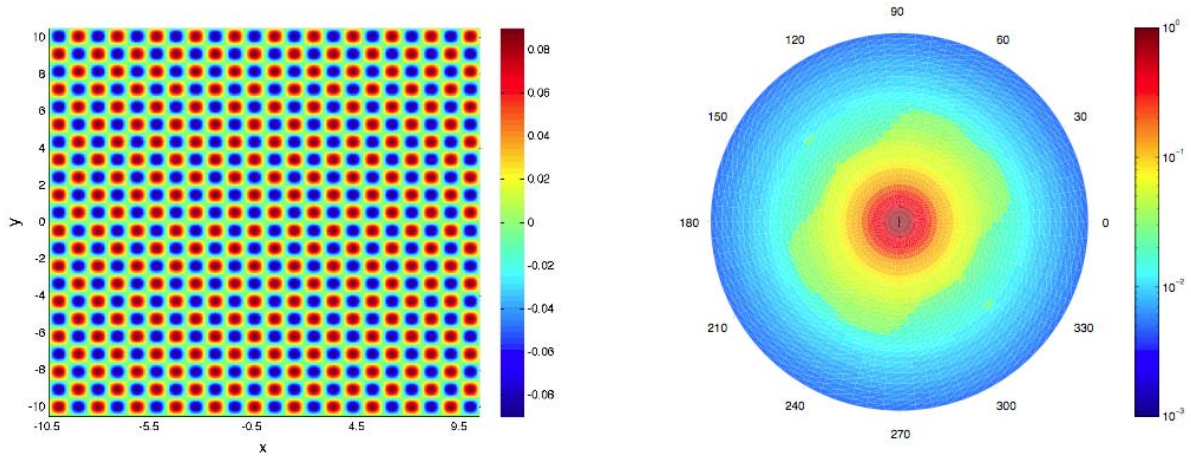
## RESULTS

a) In addition to the previous studies involving a one dimensional (1D) cosine ocean wave, we have applied the hybrid matrix operator-Monte Carlo (HMOMC) method to calculations of the downward light fields when a two dimensional (2D) sine ocean wave is present. As before, the range of the computational domain is  $-10.5 \text{ m} < x, y < 10.5 \text{ m}$  and  $0 < z < 20 \text{ m}$ . The ocean is within  $0 < z < 10 \text{ m}$  and the atmosphere is within  $10 \text{ m} < z < 20 \text{ m}$ . The extinction coefficient of the atmosphere is  $0.025 \text{ m}^{-1}$ , which means the optical depth of the atmosphere is 0.25. The Rayleigh phase matrix is chosen for the atmosphere. The extinction coefficient of the ocean is  $1.0 \text{ m}^{-1}$ , making the optical depth of the ocean 10.0. The phase function  $P_{11}$  for the ocean is chosen as the Henyey-Greenstein (H-G) phase function. The asymmetry parameter  $g = 0.95$  is used for this case. The other phase matrix elements for the ocean are determined by letting the corresponding reduced phase matrix elements be equal to those of the reduced Rayleigh phase matrix elements. The single scattering albedo is 0.5 for the atmosphere, ocean, and the Lambertian ocean bottom. The detector is located below the ocean surface with an optical depth of  $\tau = 1$  between the detector and the surface. The 2D sine ocean wave is assumed to be in the form of  $z = \sin(ax + b)\sin(cy + d)$ . We considered surface waves with various wavelengths:  $\lambda = 21 \text{ m}$ ,  $\lambda = 21/11 \text{ m}$ ,  $\lambda = 21/21 \text{ m}$ , and  $\lambda = 21/41 \text{ m}$ , or 1 wavelength, 11 wavelengths, 21 wavelengths, and 41 wavelengths in each dimension within the computational domain. The patterns of the ocean waves and the corresponding angular distributions of the downward radiance fields are shown in Figs. 2-5. In each figure, the left panel shows the pattern of the 2D sine ocean waves; the right panel shows the corresponding angular distribution of the downward radiance, which is transformed into a round disk image. For each pixel of the image, the magnitude of the radiance is shown by a color. A pixel at the center shows the radiance with a polar angle of  $180^\circ$  and a pixel at the outer edge shows the radiance with a polar angle of  $90^\circ$ . A pixel's azimuthal angle shows the corresponding azimuthal angle of the radiance.

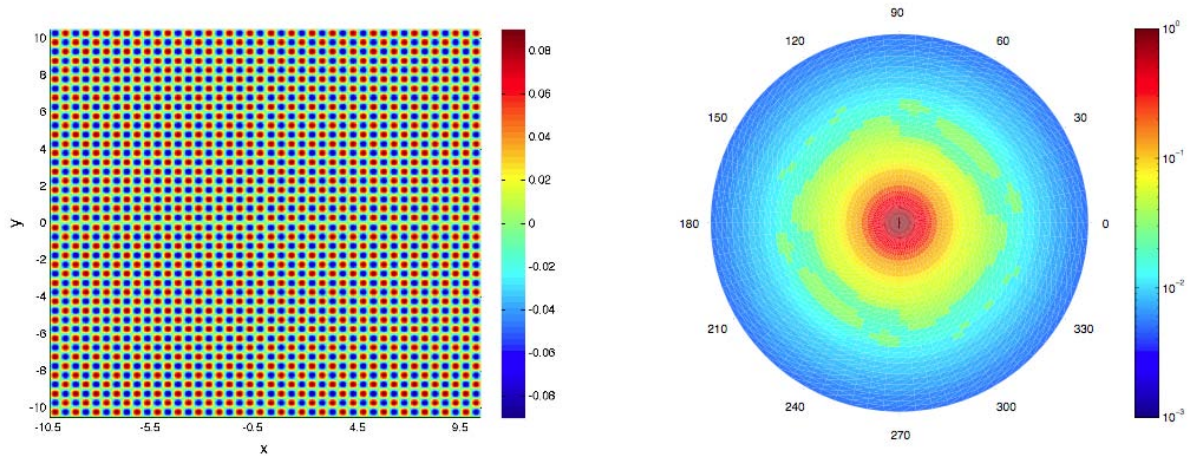


**Fig. 2.** *The left panel shows the pattern of the 2D sine ocean waves when there is one wavelength in each dimension within the computational domain. The right panel shows the corresponding angular distribution of the downward radiance.*



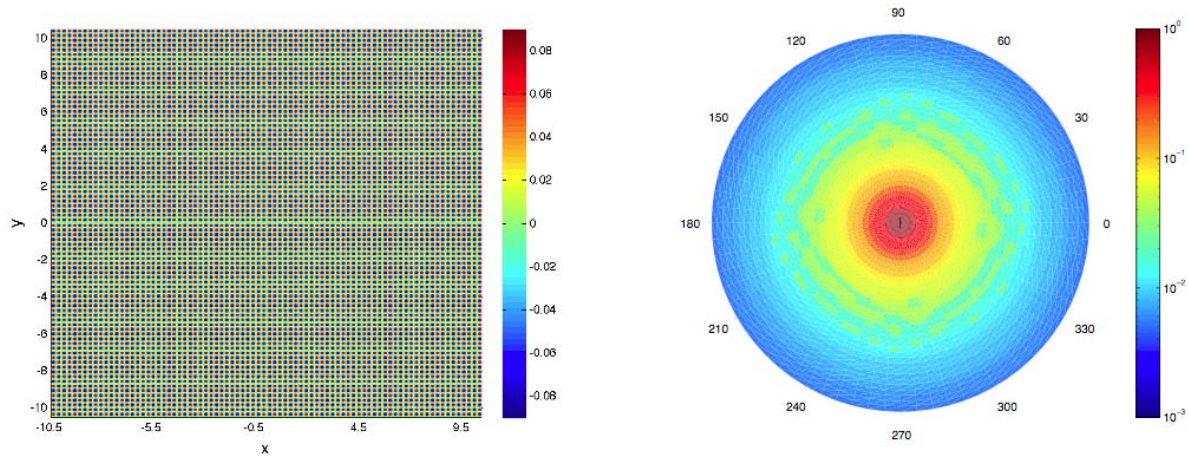


*Fig. 3. Same as Fig. 2, except that there are 11 wavelengths in each dimension within the computational domain.*



*Fig. 4. Same as Fig. 2, except that there are 21 wavelengths in each dimension within the computational domain.*





***Fig. 5. Same as Fig. 2, except that there are 41 wavelengths in each dimension within the computational domain.***

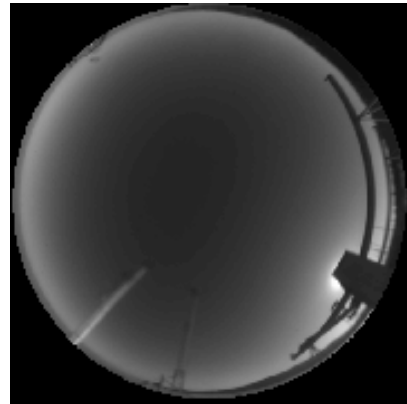
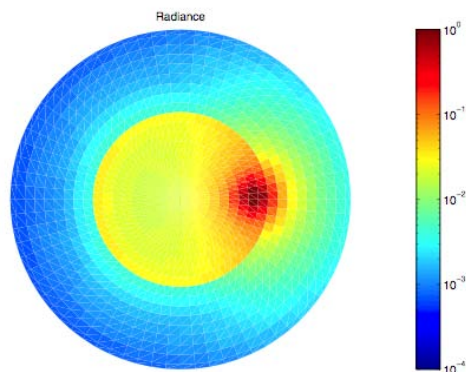
From the right panels of Figs. 6-9, it is clear that for ocean waves with wavelengths varying from half a meter to tens of meters, the angular distribution of the downward radiance looks similar. However, it is interesting to notice that as the wavelength of the ocean waves decreases, the radiance distribution has more and more fine structure. These numerical simulations suggest that we can obtain useful information on capillary waves by looking into the fine structure of the radiance distributions.

b) We have done some numerical simulations to reconstruct the sky radiance distributions measured by Dr. Ken Voss in the Department of Physics at the University of Miami. In our simulations, we assumed a solar zenith angle of 60 degrees, a conservative Rayleigh atmosphere with an optical depth of 0.25, a flat ocean surface, and an ocean with a single scattering albedo of 0.5, an optical depth of 10. The light scattering in the ocean is governed by the Heyney-Greenstein scattering phase function with an asymmetry factor of  $g = 0.95$ . The detector was located below the ocean surface with an optical depth of 1 between the detector and the surface. The simulated radiance  $I$ , Stokes parameters  $Q$ ,  $U$ , and degree of linear polarization  $P$ , as well as the counterparts from field measurements, are shown in Fig. 6.

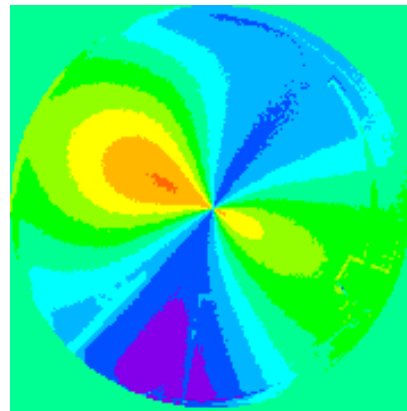
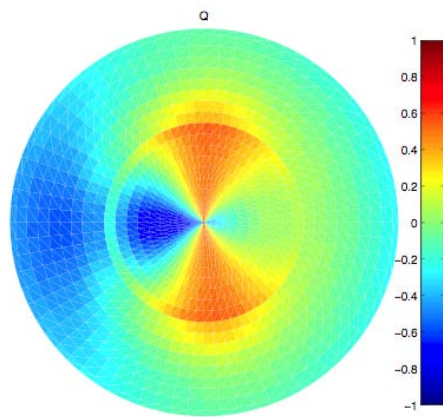
## Numerical Simulations

## Measurements

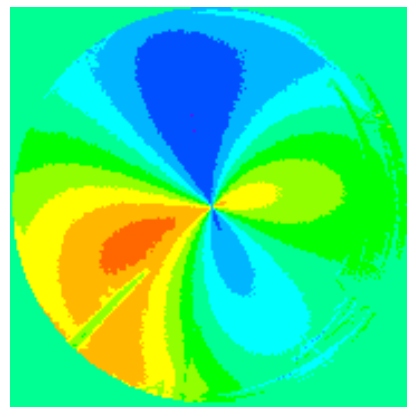
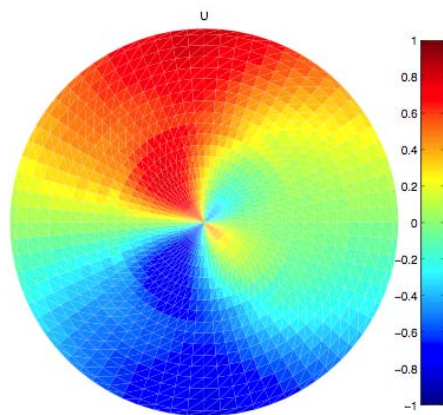
$I$

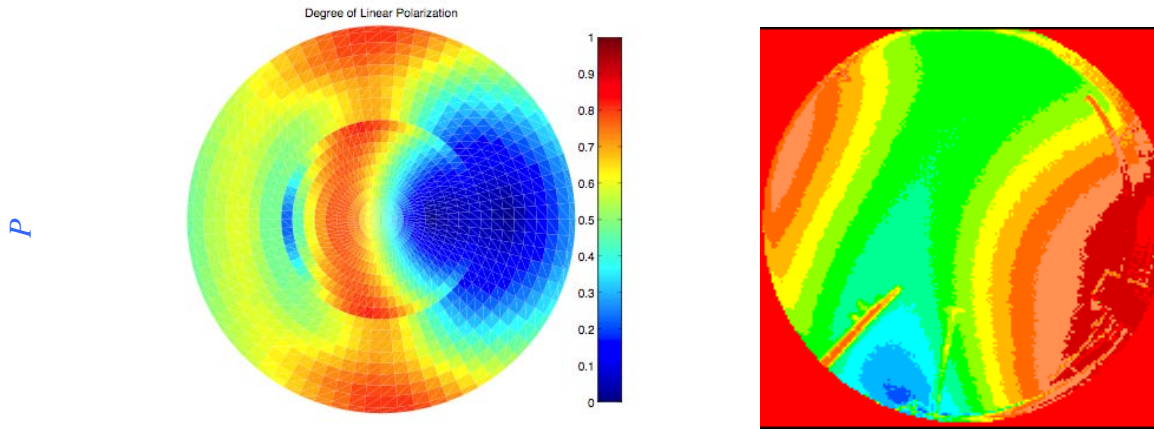


$\bar{Q}$



$U$





**Fig. 6. The angular distributions of the downward radiance ( $I$ ), Stokes parameters  $Q$  and  $U$ , and the degree of linear polarization ( $P$ ), as given by a Monte Carlo simulation (left panel) and by measurements (right panel).**

Figure 6 shows comparisons of the angular distributions of  $I$ ,  $Q$ ,  $U$ , and  $P$  given by Monte Carlo simulations compared to field measurements. It is obvious from Fig. 6 that the two sets of distributions have a good deal of similarity if the patterns given by measurement are rotated counterclockwise by an angle of 30 degrees. These comparisons imply that our Monte Carlo code is quite accurate in predicting the distribution of the polarized light field in a coupled atmosphere-ocean system. Moreover, one can notice that in the simulated results, there is a sharp circular boundary in each pattern. This is the boundary of the Snell's window, which is the conical region where the radiance directly transmitted through the surface could reach. In the outer region, on the other hand, there is only multiple-scattered radiance, which is substantially lower than the directly transmitted radiance. In the results obtained in the field measurements, however, this boundary disappears as the existence of the ocean waves blurs the images.

## IMPACT/APPLICATION

This new code together with the previous 3D Monte Carlo code for solving the vector radiative transfer equation will become powerful tools to study the effects of a dynamic wave profile and 3D inhomogeneous hydrosol distribution on the radiance field. A further study based on our current code could lead to a fundamental development of underwater image detection and remote sensing of ocean color. The methodology used in this code has been applied to model a new backscattering meter developed by Dr. Ed Fry's group. This new meter will be the only instrument ever developed to measure the backscattering coefficient directly.

## TRANSITIONS

Due to the efficiency and versatility of this new code, it will be directly applicable to understanding one of the most formidable problems in global warming, i.e. the effect of broken clouds on the reflectivity of the atmosphere. My colleague, Dr. Ping Yang in the Department of Atmospheric Sciences at TAMU, will use it to interpret the measurements of the satellites, Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observation (CALIPSO), Moderate Resolution Imaging

Spectroradiometer (MODIS), and Polarization and Directionality of the Earth's Reflectances (POLDER). This code can also be used in biomedical studies, such as numerical simulations of the light propagation in skin tissue.

## RELATED PROJECTS

We use the results from our other ONR Grant to use as input to our codes in the RaDyO study.

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